answers

Chapter 4 Review Packet

1. Write the probability rules for the following:

(a) $P(A \cap B) = P(A) \cdot P(B|A)$

(b) $P(A \cap B)$, if A and B are independent $= P(A) \cdot P(B)$

(c) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

(d) $P(A \cup B)$, if A and B are disjoint $= P(A) + P(B)$

(e) $P(B|A) = \frac{P(A \cap B)}{P(A)}$

2. Two events are independent if...
the outcome of one event has no effect on the outcome of the other event

3. Two events are disjoint if...
they cannot occur simultaneously
$P(A \cap B) = 0$

4. What is wrong with the following?

(a) The probability of Megan making it to class on time is 0.45, while the probability that she doesn't make it to class on time is twice the probability that she does. $P(\text{making to class}) = 0.45$
$P(\text{not making to class}) = 0.9$
1.35
$P(A) + P(A^c) \neq 1$

(b)

X	150	200	345	654
P(X)	0.35	0.27	0.4	0.51

$\Sigma P(x) = 1.53$ $\quad \Sigma p(x_i) \neq 1$

5. $P(A) = 0.56$, $P(B) = 0.23$, $P(A \cap B) = 0.14$

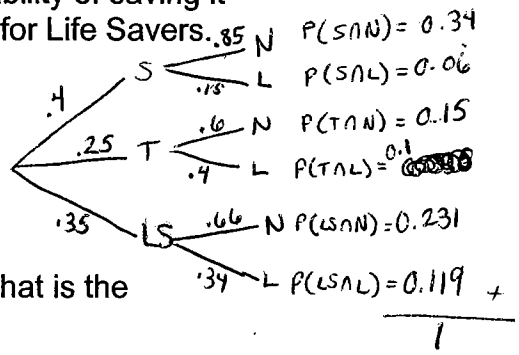
(a) What is the $P(A^c)$? $P(A^c) = 1 - P(A)$
$= 1 - 0.56 = 0.44$

(b) Are A and B disjoint? Why or why not?
NO! $P(A \cap B) \neq 0$, so they are not disjoint

(c) Decide whether A and B are independent and prove your answer
If Independent, $P(A \cap B) = P(A) \cdot P(B)$ or $P(B|A) = P(B)$
$0.14 \stackrel{?}{=} (0.56)(0.23)$
$0.14 \neq 0.1288$
NO!
$0.25 \neq 0.23$

(d) $P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{0.14}{0.56} = 0.25$

6. When a student reaches into a bag of his favorite candies, he can grab a Snickers, a Tootsie Roll, or a Lifesaver. The bag has 40% Snickers, 25% Tootsie Rolls and 35% Life Savers. After he grabs a piece of candy, he decides whether he wants to eat it now or saving it for later. The probability of saving it for later for a Snickers is 15%, 40% for Tootsie Rolls and 34% for Life Savers.



(a) What is the probability that he eats a Snickers now?

$P(S \cap N) = 0.34$

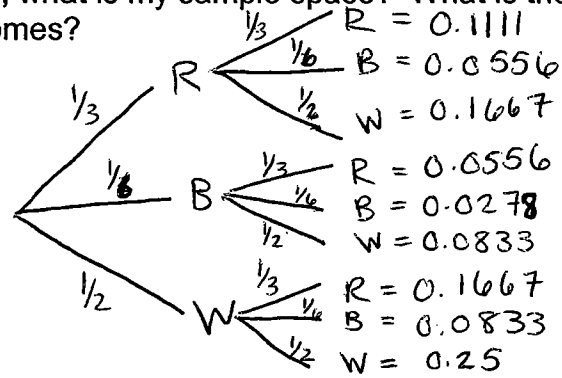
(b) Given that he is eating a piece of candy right now, what is the probability that he is eating a Tootsie Roll?

$$\frac{0.15}{0.34+0.15+0.231} = \frac{0.15}{0.721} \approx 0.208 \approx 20.8\%$$

7. I have a cup with 6 chips in it: 2 red, 1 blue, and 3 white. I want to pick out 2 chips.

$R = 1/3 \quad B = 1/6 \quad W = 1/2$

(a) If I pick a chip out, record its color, and then **replace** it before I pick the next chip, what is my sample space? What is the probability of each of my outcomes?



S =
RR WR
RB WB
RW WW
BR
BB
BW

(b) Create the probability distribution for the discrete random variable X, the number of red chips picked.

0: BB, BW, WB, WW
1: RB, RW, WR, BR
2: RR

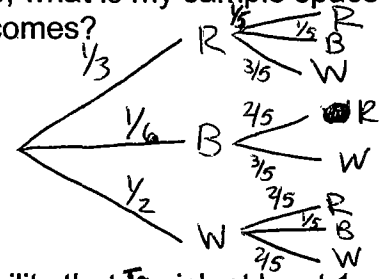
X	0	1	2
P(x)	0.4444	0.4446	0.1111

(c) What is the probability that I get at least 1 red chip? (in other words, $P(X \geq 1)$)

$0.4446 + 0.1111 = 0.5557$

$= 55.57\%$

(d) If I pick a chip out, record its color, and then **DO NOT replace** it before I pick the next chip, what is my sample space? What is the probability of each of my outcomes?



RR = 0.0667
RB = 0.0667
RW = 0.2
BR = 0.0667
BW = 0.1
WR = 0.2
WB = 0.1
WW = 0.2

(e) What is the probability that I pick at least 1 white chip?

at least 1 white: RW, BW, WR, WB, WW

$0.2 + 0.1 + 0.2 + 0.1 + 0.2 = 0.8 = 80\%$

8. One thousand students at a city high school were classified both according to GPA and whether or not they consistently skipped classes.

	GPA <2.0	GPA 2.0-3.0	GPA >3.0	
Many Skipped	80	25	5	110
Few Skipped	175	450	265	890
	255	475	270	1000

a. What is the probability that a student has a GPA between 2.0 and 3.0?

$475/1000 = 0.475$

b. What is the probability that a student has a GPA under 2.0 or has skipped many classes?

$\frac{80 + 175 + 25 + 5}{1000} = 0.285$

c. What is the probability that a student has a GPA under 2.0 given that s/he has skipped many classes?

$\frac{80}{110} \approx 0.727$

9. An experiment consists of selecting one card from a deck. Based on events A, B and C below, calculate the following probabilities:

A = selecting a king $= 4/52 = 1/13$
B = selecting a club $= 13/52 = 1/4$
C = selecting a red card $= 26/52 = 1/2$

$P(B|A) = 1/4$ ↑ P(if I select a king, it will be a club)

$P(C|B) = 0$

$P(A \cap C) = 2/52 = 1/26$

(a) $P(A \cap B)$

$P(A \cap B) = P(A) \cdot P(B|A)$

$= 1/13 \cdot 1/4 \approx 0.1923 \; (= 1/52)$

(b) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$= 1/13 + 1/4 - 1/52 = 4/13 \approx 0.3077$

(c) $P(A \cup C) = P(A) + P(C) - P(A \cap C)$
$= 1/13 + 1/2 - 1/26 = 7/13 \approx 0.5385$

(d) $P(B \cap C) = P(B)\,P(C|B) = 0$

(e) $P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{1/52}{1/4} = 1/13 \approx 0.0769$

10. For the probability distribution below, find the expected value, the variance, and the standard deviation

X	10	25	40	55
P(X)	0.21	0.17	0.28	0.34

$E(X) = (10)(0.21) + (25)(0.17) + (40)(0.28) + (55)(0.34)$
$= 36.25$
$\sigma = 17.0202$
$\sigma^2 = 289.687$

11. I am in a drawing at the carnival. There are 5 first place prizes of $100, 10 second place prized of $50, and 20 door prizes, each worth $5. The carnival workers have sold 1,200 tickets. What is the expected winning for this drawing?

X	0	5	50	100
P(x)	1165/1200	20/1200	10/1200	5/1200

$E(X) = (0)\left(\frac{1165}{1200}\right) + (5)\left(\frac{20}{1200}\right) + (50)\left(\frac{10}{1200}\right) + (100)\left(\frac{5}{1200}\right)$
$\approx \$0.92$

12. Suppose that, in a certain part of the world, in any 50-year period the probability of a major plague is .39, the probability of a major famine is .52, and the probability of both a plague and a famine is .15. What is the probability of a famine given that there is a plague?

A. .240 B. .288 C. .370 D. .385 E. .760

$P(P) = 0.39$
$P(F) = 0.52$
$P(P \cap F) = 0.15$

$P(F|P) = \frac{P(F \cap P)}{P(P)} = \frac{0.15}{0.39} \approx 0.385$

13. Below are the probability distributions for random variables X and Y. Assuming X and Y are independent, create their joint probability table

X	P(X)
1	0.37
2	0.25
3	0.38

Y	P(Y)
1	0.42
2	0.13
3	0.27
4	0.18

X \ Y	1	2	3	4	
1	0.1554	0.0481	0.0999	0.0666	0.37
2	0.105	0.0325	0.0675	0.045	0.25
3	0.1596	0.0494	0.1026	0.0684	0.38
	0.42	0.13	0.27	0.18	1

14. Below is the joint probability table for random variables X and Y. Create the individual probability distributions for X and Y.

		Y			
		1	2	3	
X	1	0.04	0.06	0.1	0.2
	2	0.15	0.07	0.08	.3
	3	0.21	0.13	0.16	.5
		0.4	0.26	0.34	

X	P(X)
1	0.2
2	0.3
3	0.5

Y	P(Y)
1	0.4
2	0.26
3	0.34

15. The mean and variances of discrete random variables T and N are given below. Find each of the following:

$\mu_T = 45.3$ $\quad$ $\mu_N = 23.8$
$\sigma^2_T = 3.1$ $\quad$ $\sigma^2_N = 2.4$

(a) μ_{3T}
(b) $\sigma^2_{2+0.5N}$
(c) μ_{T+N}
(d) σ^2_{3T+2N}
(e) μ_{-2+2N}
(f) σ_{4T+5}

a) $\mu_{3T} = 3\mu_T = 135.9$

b) $\sigma_{2+0.5N} = (0.5)^2(\sigma_N^2) = 0.6$

c) $\mu_{T+N} = \mu_T + \mu_N = 69.1$

d) $\sigma^2_{3T+2N} = \sigma^2_{3T} + \sigma^2_{2N} = 3^2\sigma_T^2 + 2^2\sigma_N^2$
$= 9 \cdot 3.1 + 4 \cdot 2.4$
$= 37.5$

e) $\mu_{-2+2N} = -2 + 2\mu_N = 45.6$

f) $\sigma_{4T+5} = \sqrt{\sigma^2_{4T+5}} = \sqrt{4^2 \cdot \sigma_T^2} = \sqrt{16 \cdot 3.1}$
$= \sqrt{49.6} \approx 7.043$